ECON 4925 Autumn 2011
Resource Economics

Water as a natural resource

Førsund (2007), Chapter 2

Lecturer:

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Economic use of water

- Rivers, lakes, groundwater
- Fish, transport, drinking water households, process water industry, irrigation agriculture, hydropower
- Hydropower: converting energy in falling water to electricity
  - Run of the river
  - Reservoirs

Many user groups

- Utility functions in water

$$U_t^g(r_t^g)\ ,\ U_t^{g\prime} \geq 0, U_t^{g\prime\prime} \leq 0,\ \ g=1,..,G,\ t=1,..,T$$

- Availability constraint for water

$$\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g \leq W$$

- The social planning problem

$$\max \sum_{t=1}^{T} B(U_t^1(r_t^1),..,U_t^n(r_t^G)) \text{ subject to}$$

$$\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g \leq W$$

$$r_t^g \geq 0, g=1,..,G, t=1,..,T; T, W \text{ given}$$

- The Lagrangian

$$L = \sum_{t=1}^{T} B(U_t^1(r_t^1),..,U_t^G(r_t^G)) - \lambda(\sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g - W)$$

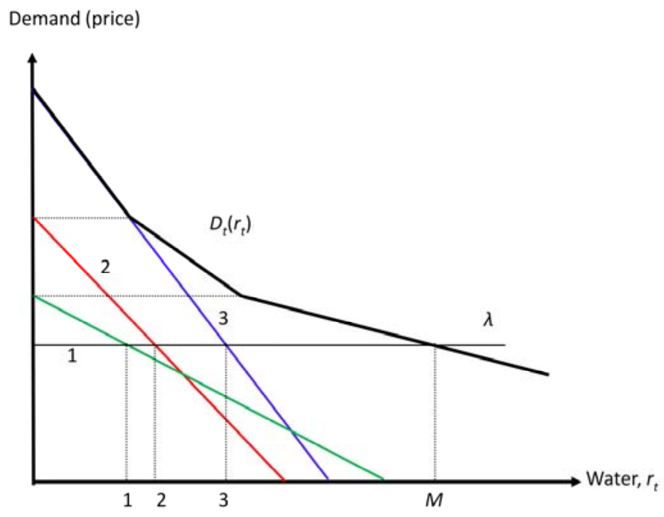
- Necessary first-order conditions

$$\frac{\partial L}{\partial r_t^g} = B_g{}' U_t^{g}{}'(r_t^g) - \lambda \le 0 \; (= 0 \text{ for } r_t^g > 0), t = 1,..,T, g = 1,..,G$$

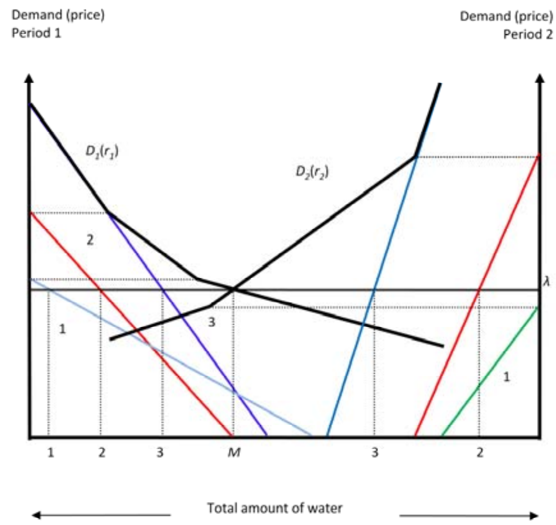
$$\lambda \ge 0 \; (= 0 \text{ for} \sum_{t=1}^{T}\sum_{g=1}^{G} r_t^g < W)$$

  – Socially weighted marginal utilities of water should all be equal between different user groups and equal over time, and equal to the water value

Aggregation of group demands



Allocation over time



The basic hydropower model

- Hydropower plants aggregated to one system
- Reservoir, stock of water R
- Inflows w and releases r
- The dynamics of water accumulation: $R_t = R_{t-1} + w_t - r_t$, t=1,..,T
  - Inequality implies overflow
- Converting water from the dam to electricity $e^H_t = (1/a)r_t$
- Fabrication coefficient, a, assumed to be constant

The basic hydropower model, cont.

- Assuming a dominating period of inflows: all inflows come in period 1:

$$\sum_{t=1}^{T} r_t = w_1 \Rightarrow \sum_{t=1}^{T} a e_t^H = w_1$$

- Converting water to units of kWh:

$$\sum_{t=1}^{T} e_t^H = \frac{w_1}{a} = W$$

- Assuming that the reservoir will never be full
- Perfect transferability of water between periods up to the horizon T (start of new cycle)

Social evaluation of electricity

- Evaluation of electricity by utility functions

$$U_t(e_t^H) \ , \ U_t'(e_t^H) \geq 0 \ , \ U_t''(e_t^H) < 0 \ , \quad t = 1,..,T$$

- Measuring utility in money
- Marginal willingness to pay and the demand function on price form

$$U_t'(e_t^H) \equiv p_t(e_t^H)$$

The social planning problem

- Social planner's optimisation problem

$$Max \sum_{t=1}^{T} U_t(e_t^H)$$

$$s.t.$$

$$\sum_{t=1}^{T} e_t^H \le W, e_t^H \ge 0, t = 1,..,T$$

$$W, T \text{ given}$$

- The Lagrangian

$$L = \sum_{t=1}^{T} U_t(e_t^H) - \lambda(\sum_{t=1}^{T} e_t^H - W)$$

The solution

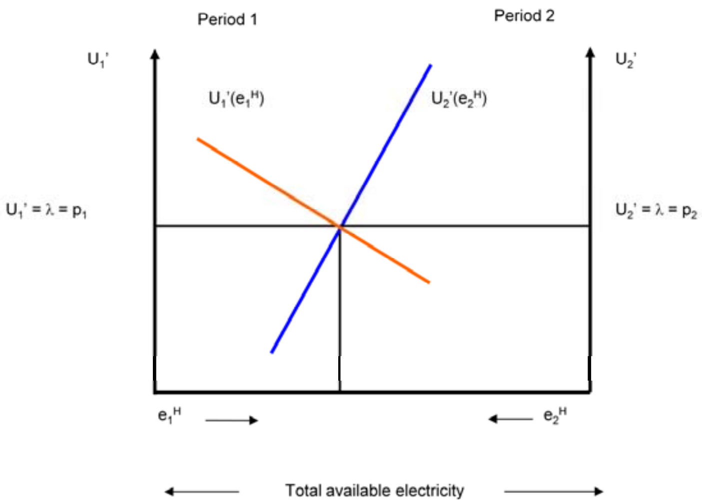
- Necessary first-order conditions

$$\frac{\partial L}{\partial e_t^H} = U_t'(e_t^H) - \lambda \leq 0 \; (= 0 \text{ for } e_t^H > 0), t = 1,..,T$$

$$\lambda \geq 0 \; (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W)$$

- Shadow price on water is zero if not all water is used
- Marginal willingness to pay, i.e. social price, equal for all periods

The two-period bathtub illustration



Introducing discounting

- Discount factor: $\beta_t = (1+r)^{-(t-1)},\ t = 1,..,T$
- The optimisation problem

$$Max \sum_{t=1}^{T} U_t(e_t^H)\beta_t \ s.t.$$

$$\sum_{t=1}^{T} e_t^H \le W$$

- Necessary first–order conditions

$$\frac{\partial L}{\partial e_t^H} = U_t'(e_t^H)\beta_t - \lambda \le 0 \ (= 0 \text{ if } e_t^H > 0), t = 1,..,T$$

$$\lambda \ge 0 \ (= 0 \text{ if } \sum_{t=1}^{T} e_t^H < W)$$

The Hotelling rule

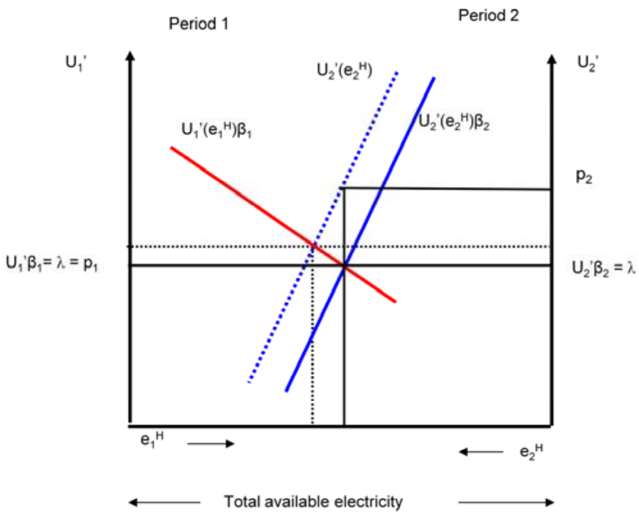
- Growth rate in marginal utility

$$U_t'(e_t^H)\beta_t = U_{t+1}'(e_{t+1}^H)\beta_{t+1} \Rightarrow U_{t+1}'(e_{t+1}^H) = U_t'(e_t^H)\frac{\beta_t}{\beta_{t+1}} = U_t'(e_t^H)(1+r)$$

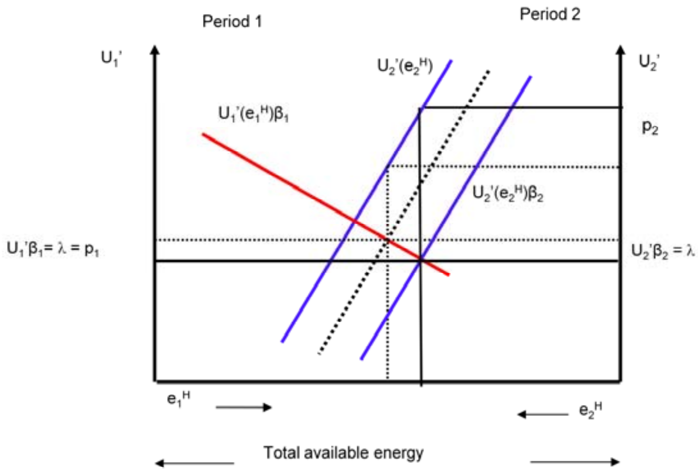
$$\frac{U_{t+1}'(e_{t+1}^H) - U_t'(e_t^H)}{U_t'(e_t^H)} = \frac{U_t'(e_t^H)(1+r) - U_t'(e_t^H)}{U_t'(e_t^H)} = r$$

$$\frac{p_{t+1}(e_{t+1}^H) - p_t(e_t^H)}{p_t(e_t^H)} = r$$

The effect of discounting



The effect of increased discount rate



The effect of more water